Knife Edge Scanning for Beam Profiling

The power you measure from a laser beam on a power meter is given by:

$$P = \int_{\infty}\int_{\infty} I(x,y)\mathrm{d}x\mathrm{d}y \tag{1}$$

Where $I(x,y)$ is the intensity of the beam in the x and y directions. If the beam is a Gaussian beam (probably is), then you can write this as:

$$P = I_0 \int_{-\infty}^{\infty} \exp\bigg(-\frac{2x^2}{\omega_x^2}\bigg)\mathrm{d}x \int_{-\infty}^{\infty} \exp\bigg(-\frac{2y^2}{\omega_y^2}\bigg)\mathrm{d}y \tag{2}$$

Where I_0 is the incident intensity, and ω_x is the $1/\mathrm{e}^2$ beam **radius** in the x direction. This is one of many different ways to define the size of a laser beam, but is the one people probably use most. It's the radius at which the intensity has fallen to $1/\mathrm{e}^2$[1].

[1]Or when the electric field amplitude has fallen to $1/\mathrm{e}$.

If we scan a knife edge across the face of the beam whilst measuring the power behind the knife edge, then the power will drop as the knife edge blocks the beam. This is a quick way to get an idea of the size of the beam, as you can relate the measured power as a function of the knife edge position to the beam radius ω. The program linked to this does this fit, but the equation fitted to is derived as follows.

If we scan a knife edge into the beam in a way that it only is moving along x or y (easy as we are free to define these coordinates anyway). Then we can say that power when the knife edge is at a position D in the beam is given by:

$$P(D) = I_0 \int_{D}^{\infty} \exp\bigg(-\frac{2x^2}{\omega_x^2}\bigg)\mathrm{d}x \int_{-\infty}^{\infty} \exp\bigg(-\frac{2y^2}{\omega_y^2}\bigg)\mathrm{d}y \tag{3}$$

Where we scan it along x, but the same equation holds for y as we can swap the integration variables at leisure. We assume that we are bringing the knife edge in from one direction, but this doesn't really matter in the end. Doing these integrals leads to:

$$P(D) = \frac{I_0\omega_x\omega_y\pi}{4}\mathrm{erfc}\bigg(\frac{D\sqrt{2}}{\omega_x}\bigg) \tag{4}$$

Where erfc is the complementary error function, which is $1-\mathrm{erf}$ where $mathrmerf$ normal error function, which is what you get if you integrate this Gaussian.

However, it is easier for us to use this expression if we divide it by the maximum possible transmitted power, P_{inf}, if we have no knife edge in the way:

$$\frac{P(D)}{P_{\mathrm{inf}}} = \frac{1}{2}\mathrm{erfc}\bigg(\frac{D\sqrt{2}}{\omega}\bigg) \tag{5}$$

Where I removed the subscript from the beam radius, to reflect that it doesn't matter which one you measure. If you scan the knife edge along x, that the one you measure and vice versa (I think this is obvious).

The final point is that this derivation has assumed that our coordinate system is centered at the middle of the beam, which it probably isn't unless you set the micrometer stage really well. So actually we can just write:

$$\frac{P(D)}{P_{\mathrm{inf}}} = \frac{1}{2}\mathrm{erfc}\left(\frac{(D - D_0)\sqrt{2}}{\omega}\right) \tag{6}$$

Where D_0 is the position of the knife edge at the middle of the beam. This is the equation the fitting program fits to. All you need to do is measure the power of the beam as a function of the stage position!